1. You are given the statistic \( s \) for a training set \( \mathcal{X} = \{x_1, \ldots, x_n\} \),
\[
s = \frac{1}{n} \sum_{i=1}^{n} x_i
\]
where \( x_i \) are discrete-valued scalars. You see only \( s \), and not the specifics of \( \mathcal{X} \).

(a) Give three examples of distribution assumptions under which this statistic is sufficient for an unknown parameter. (See Table 3.1 from DHS, posted with this assignment.)

(b) Choose one of the cases you named and find the MAP estimate of the unknown parameter \( \theta \) from \( s \) using an exponential distribution with parameter \( \alpha \) as a prior.

(c) Is it possible to find \( p(x|\mathcal{X}) \) given only \( s \) when \( s \) is sufficient for \( \theta \) in \( p(x|\theta) \)? Explain.

2. Write a program (e.g. Matlab) to estimate the mean and covariance of the 3-dimensional Gaussian distribution using the data on the handout web page. Use the EM algorithm to deal with the missing data (indicated by a very large number) with 10 iterations. Turn in your code, the parameters that you estimated and the likelihood of the observed data in each iteration. To compute the likelihood of the observed data, you should marginalize out the “missing” elements.

3. Derive the E and M update steps for estimating the parameters of a mixture distribution
\[
p(x) = \sum_{k=1}^{K} \lambda_k p_k(x),
\]
where the mixture components \( p_k(x) \) are Rayleigh distributions with parameter \( \theta_k \).