EE512 Graphical Models	**University of Washington**
Spring 2006	**Dept. of Electrical Engineering**

Homework 4: Due Friday, May 5, 2006, 5:00pm

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Problem 1. Phylogenetic Trees 1

Draw the graph G_T and phylogenetic tree for the characters $C = \{a, b, c\}$, and the following set of 4 species:

a a a
b c a
b b b
c a b

Problem 2. Phylogenetic Trees 2

Give an example of a set of species and characters that does not have a perfect phylogeny.

Problem 3. Triangulation Heuristics

The following web page contains definitions for 2 graphs:

`http://ssli.ee.washington.edu/courses/ee512/hw4/hw4_files.html`

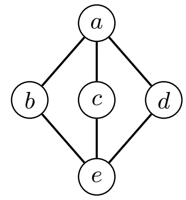
For each graph $G = (V(G), E(G))$, find a triangulation of G, denoted as $T(G) = (V(T(G)), E(T(G)))$, with the smallest possible maximum clique size (i.e., your resulting triangulations should be such that the largest clique has as few variables as possible, but it should still be the case that $V(G) = V(T(G))$ and that $E(G) \subseteq E(T(G))$). Recall that we call $E(T(G)) \setminus E(G)$ the *fill-in* edges. Email your code, solutions, and a brief desription of the method(s) you used to the TA (`bartels@ee.washington.edu`). Use any programming language you are comfortable with (except cobol).

The triangulations should be output into a file by listing one edge per line. For each graph, include both the orginal graph edges $E(G)$ and fill-in edges $E(T(G)) \setminus E(G)$. For example, the following defines a triangulated graph with four vertices and five edges:

v1, v2
v2, v4
v3, v2
v4, v3
v4, v1

Problem 4. Inference

Write a program to find the marginal probability of e in the following system. Points will be given for doing this efficiently (in a order of magnitude sense). Your program does not need to be general purpose, it should be designed to find $p(e)$. Email your code along with your answer to the TA (`bartels@ee.washington.edu`). Use any language you are comfortable with.



The joint distribution is given as follows:

$$p(a,b,c,d,e) = \frac{1}{Z}\prod_{i=1}^{6}\psi_i(x_{C_1})$$

The general form of the potential function is as follows

$$\psi(a,b) = \begin{pmatrix} \psi(a=1,b=1) & \psi(a=1,b=2) \\ \psi(a=2,b=1) & \psi(a=2,b=2) \end{pmatrix}$$

where the actual potential for each of the edges in the above graph are given as the following:

$$\psi_1(a,b) = \begin{pmatrix} 1 & 2 \\ 3 & 4 \end{pmatrix} \quad \psi_2(a,c) = \begin{pmatrix} 5 & 6 \\ 7 & 8 \end{pmatrix} \quad \psi_3(a,d) = \begin{pmatrix} 9 & 10 \\ 1 & 2 \end{pmatrix}$$

$$\psi_4(b,e) = \begin{pmatrix} 3 & 4 \\ 5 & 6 \end{pmatrix} \quad \psi_5(c,e) = \begin{pmatrix} 7 & 8 \\ 9 & 10 \end{pmatrix} \quad \psi_6(d,e) = \begin{pmatrix} 1 & 2 \\ 3 & 4 \end{pmatrix}$$

$$Z = \frac{1}{211446}$$

Problem 5. More Inference

Write a program to find the assignment for variables a, b, c, d, e that gives the highest probability for the system in problem 4.

Hint: this will only require small changes to your solution to problem 4

Problem 6. Minimal separators

5a) In a general graph G, can one minimal separator be properly contained in another minimal separator? In other words, lets suppose that S_1 and S_2 are minimal separators, with $S_i \subseteq V$, for graph $G = (V, E)$. Is it possible that we could have $S_1 \subset S_2$? If so, give an example graph and clearly indicate the minimal separators, and if not, prove that it can not.

5b) Repeat the same question, but in this second case you assume that the graph G is triangulated.

Problem 7. Inclusion-Exclusion

Let V be a finite set, and $A_i \subseteq V$ for all i. Recall the general form of inclusion-exclusion on set sizes.

$$|A_1 \cup A_2 \cup \ldots \cup A_k| = \sum_{1\le i\le k}|A_i| - \sum_{1\le i_1<i_2\le k}|A_{i_1}\cap A_{i_2}| + \sum_{1\le i_1<i_2<i_3\le k}|A_{i_1}\cap A_{i_2}\cap A_{i_3}| - \ldots + (-1)^{k-1}|A_1\cap A_2\cap\ldots\cap A_k|$$

Prove this general formula using Möbius inversion lemma. This means that you'll need to come up with valid function forms for both ϕ and ψ, and clearly demonstrate why these are valid.

Problem 8. Bayesian networks that when moralized become undirected trees

In class, we shows that the alpha/beta recursion for HMMs was that $\alpha_t(j) = p(x_{1:t}, Q_t = j)$ and $\beta_t(j) = p(x_{t+1:T}|Q_t = j)$. We also saw the lambda/pi recursion in trees, where $\lambda(x) = p(\bar{z}_E|x)$ and $\pi(x) = p(x|\bar{y}_E)$.

7a) First, show that while these recursions are similar, they are not exactly the same when the tree is an HMM (where we use the typical evidence pattern for an HMM, namely where Q are the hidden variables and X are the observed variables).

7b) Come up with a recursion for BN trees (not polytrees, just regular trees) that is exactly the same as the alpha/beta recursions for HMMs when the tree happens to be an HMM, but the recursions should still work for all trees, when the trees are not an HMM (or the evidence pattern is not the same as we typically find in an HMM).