Probability and Statistics for Engineers
IndE 315

Unit 3: Statistical Inference

Nov. 10, 2010

Announcements

Guest lecturer

- Julie Medero, MSEE (Dec.)
  - researcher in Electrical Engineering
  - studying statistical language processing
  - computer science and linguistics background

Topic: Type 1 and Type II errors, Power

- material covered will be on Exam 3 (9-1.2)

Today's class topics

- **Null Hypothesis**
- Type I and Type II errors
- Power of a test
- Sample size

- Next assignments
- Class exercise

Null Hypothesis

- Before we talk about accepting/rejecting it, what is it?

Null Hypothesis

- What is it?
  - A (testable) statement about a parameter
  - e.g. "The mean is 10 units."
- Where does it come from?
  - Past experience
  - Theory or modeling
  - Specifications or contractual obligations

Alternative Hypothesis

- Statement that conflicts with the null hypothesis
  - One sided: "The mean > 10 units"
  - Two sided: "The mean ≠ 10 units"
- Frequently what we're actually interested in detecting
  - "The patient has cancer"
  - "The iPod is malfunctioning"

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Reading Q2: Type II error

What is a Type I error? What is a Type II error?
“Case A is Type II error because it occurs when the given parameter actually isn't true but is accepted. Therefore, Type II error is more critical because in Case A, the company would be shipping a product that actually shouldn't be shipped, while in Case B, the company is being safe in deciding not to ship, which is Type I error.”
“Case A, because according to the text, "Failing to reject the null hypothesis when it is false is defined as a type II error." So, this means that we would be shipping bad iPods, even though our sample mean gave us the impression that they were fine.”
– *student responses*

Type I and Type II errors

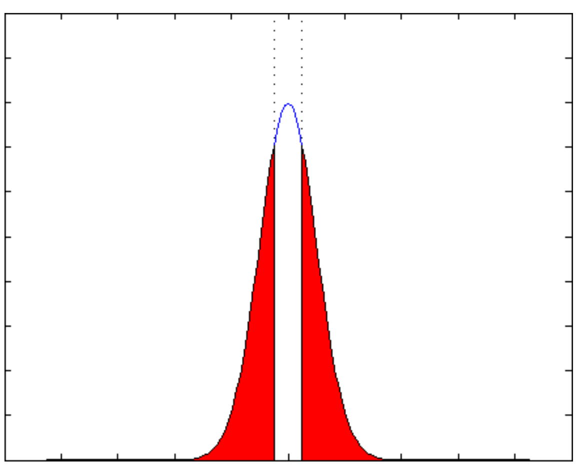
Decision	H_0 is True	H_0 is False
P(Fail to reject H_0)	No Error	Type II Error
P(Reject H_0)	Type I Error	No Error

Type I and Type II errors

- Type I error
  - Rejecting H_0 when it is true
  - AKA "false positive," "false alarm," or "producer's error"
  - P(Type I error) = α
  - Probability of thinking something's out of the ordinary when it's not.

Type I and Type II errors

- What is needed to characterize Type 1 Errors?
  - Range around the assumed (null hypothesis) value that is considered "close enough"
    - Values outside of this are critical region
    - Bigger range ➔ Lower Error



Type I and Type II errors

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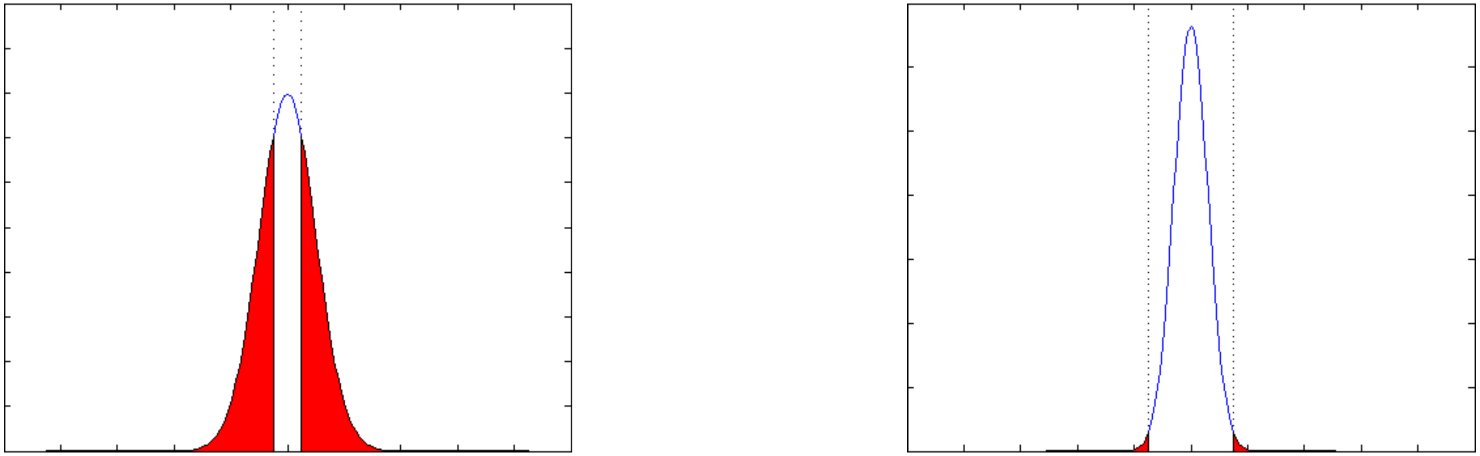
    - Values outside of this are critical region
    - Bigger range ➔ Lower Error
  - Sample Size
    - More samples ➔ Lower Error for fixed critical region

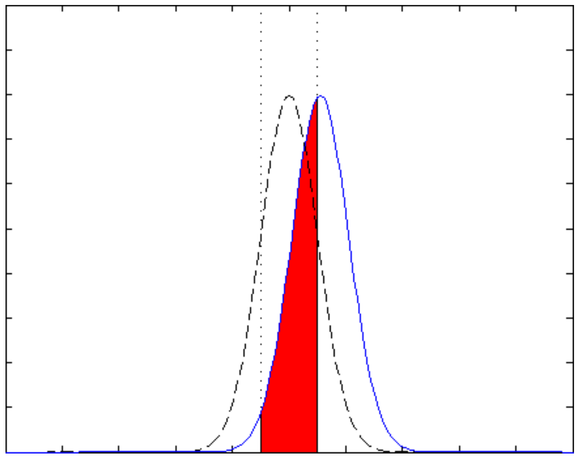


Type I and Type II errors

- Type II error
  - Failing to reject H_0 when it is false
  - AKA "false negative," "miss," or "consumer's error"
  - P(Type II error) = β
  - Probability thinking everything's ordinary when it's not.

Type I and Type II errors

- What is needed to characterize Type II Errors?
  - The actual mean
    - Farther from μ ➔ Lower Error



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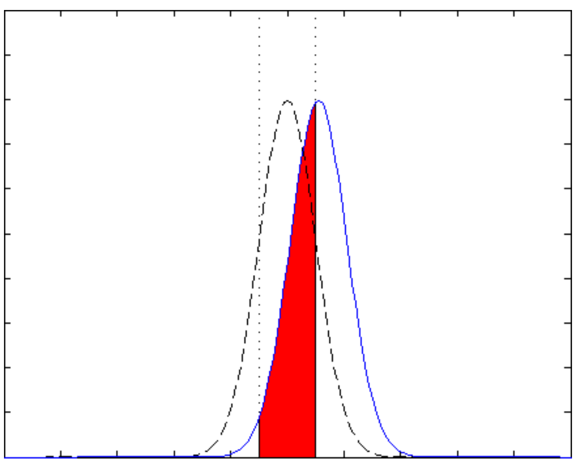

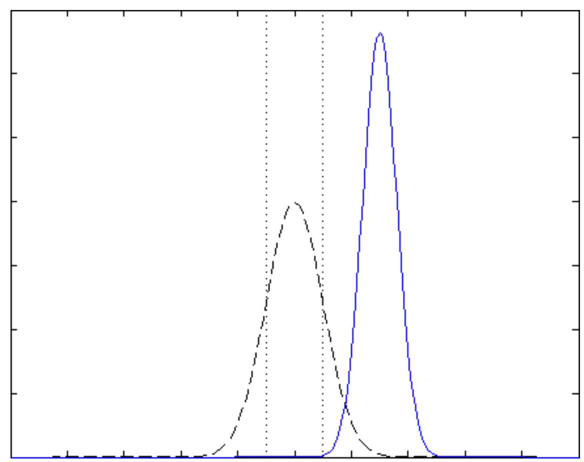

Today's class topics

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- Type I and Type II errors
- **Power of a test**
- Sample size

- Next assignments
- Class exercise

Power of a test

- Power of a statistical test
  - P(rejecting H_0 when alternative hypothesis is true) or
  - P(correctly rejecting a false null hypothesis)
  - How likely you are to recognize when things are out of the ordinary.
  - Power = 1-β

Decision	H_0 is True	H_0 is False
P(Fail to reject H_0)	1-α (Specificity)	β
P(Reject H_0)	α	1-β (Power)

Today's class topics

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- **Sample size**

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Sample size

- Only way to reduce Type I and Type II errors!
- What limits it?
  - Time
  - Cost
  - Trade-off between cost an error rate

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- **Next assignments**
- Class exercise

Next assignments

- Friday 11/12:
  - Reading response due
  - Sections 9-4, 9-5 and 8-6 (prediction, tolerance intervals)
- Monday 11/15:
  - Reading response due
  - HW #6 due
  - Quiz #6

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- Null Hypothesis
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- Next assignments
- **Class exercise**

In-class Exercise

- In-class exercise
  - Type I and Type II errors
  - Error Costs

  - **limited time**
  - **check answers in the full class**
  - **for credit, log onto course website and answer question**

In-class Exercise

Suppose our company will be fined if the widgets we produce have a size distribution with $\mu < 100$ units. Our QA department has the capacity to test 100 samples from each day's production.

- What is H_0?
- What is H_1?
- One-sided or two-sided?

In-class Exercise

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- What is H_0?
  - μ=100 units
- What is H_1?
  - $\mu < 100$ units
- One-sided or two-sided?
  - One-sided

In-class Exercise

H_0: $\mu=100$; H_1: $\mu<100$; n=100. Now suppose that we know from experience that $\sigma = 10$ units, and that from time to time one of our machines gets out of alignment and we produce widgets with $\mu=96$ units.

Critical Value	α	β
99		
98.5		
98		
97.5		
97		

In-class Exercise

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Critical Value	α	β
99	0.1587	0.0013
98.5	0.0668	0.0062
98	0.0228	0.0228
97.5	0.0062	0.0668
97	0.0013	0.1587

In-class Exercise

H_0: μ=100; H_1: μ<100; n=100; σ = 10; actual μ=96 units on broken days.

If Type II errors cost us 10 times as much as Type I errors, which critical value would minimize error-related costs?

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In-class Exercise

H_0: μ=100; H_1: μ<100; n=100; σ = 10; actual μ=96 units on broken days. If Type II errors cost us 10 times as much as Type I errors, minimum cost with critical value set to 98.5.

What else might we want to know if we wanted to minimize overall cost to the factory?

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